Indian Statistical Institute
M.Math I Year
First Semester Back Paper Examination, 2005-2006
General Topology

Time: 3 hrs Date: -12-05

Attempt all questions. All questions carry equal marks. Any result proved in the class may be cited and used without proof.

1. Let X be a topological space such that every real valued function on X is continuous. Determine the topology on X.
2. Let $F \subseteq \mathbb{R}^n$ be a closed subspace. Prove or disprove: F is connected if and only if F is path connected (give a proof if true or a counterexample if false).
3. Let X, Y be topological spaces, $f : X \to Y$ be a continuous map having a continuous section $s : Y \to X$ i.e., $f \circ s = 1_Y$. Prove that f is a quotient map.
4. a) Let X be a topological space. Prove that every path connected subspace of X is contained in a unique path component of X.

   b) Let $\pi_0(X)$ denote the set of path components of X. For $f : X \to Y$ continuous, let $\pi_0(f) : \pi_0(X) \to \pi_0(Y)$ be the function that maps a path component C of X to the unique path component of Y that contains $f(C)$. Let $g : X \to Y$ be continuous. Show that if $f \simeq g$ then $\pi_0(f) = \pi_0(g)$.
5. Prove or disprove: $\mathcal{S}^1 \times \mathcal{S}^1 \times \mathcal{S}^1$ is homotopically equivalent to $\mathcal{S}^2 \times \mathcal{S}^1$. (give a proof if true, a counterexample if false).